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ME 4133, Fall 2011

Project 2: Velocity and Acceleration Analysis

November 4, 2011

Project 2 Grade Sheet

Please print out and attach this page to your project 2

1) The project report must be enclosed in a folder.
2) The project report must be ordered in the same requirements as project 1

ID:	Points	
Flow Chart:	5	
Report: Organization	5	
Grammar and Spelling	5	
Problem Definition	5	
Methods	10	
Results/Proof of Success	10	
Program: Standard header and comment	5	
Structure	10	
Interactive Input of the parameters (Interval, Velocity, Acceleration)	10	
Calculation: The values of t2, t3,t4,w3,w4,a3,a4	10	
Plotting: Position figure	5	
Velocity figure	5	
Acceleration figure	5	
Test Program: Submit the program to the TA to test	10	
Bonus: Use subplot to display multiple plots per figure (5); Annotation plot: titles, labels, legends, and mathematical symbols (5)	10	
TOTAL	100(110)	

Reminder: Some students forgot to attach the grade sheet in project 1. This time there will be a -5 points toward your total grades if you fail to attach the grade sheet

Velocity and Acceleration Analysis of a Four Bar Linkage

ME 4133 Project 2

Kori Lutenbacher

ABSTRACT

Four bar linkages are the building blocks of understanding mechanical design and more complex mechanisms. By modeling the motion of a crank-rocker based on the given design specifications and constraints, the overall purpose and function of the mechanism can be identified. By performing Jacobian matrices and derivations, the pertinent equations were generated for analyzing the four bar mechanism's displacement, angular velocity, and angular acceleration. The program proved successful and matched well with theoretical knowledge.

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Problem Statement

Four bar mechanisms are the simplest form of any mechanical device. Being able to fully characterize their motion can help build a strong foundation for mechanical design. By determining a mechanism's function can give a lot of information on the mechanism's structure. Going back to the fundamentals assists in the design process.

Characterizing between rotational and translational motion and determining the lengths of the links help categorize the mechanism. The designs constraints are the first link is fixed (ground) at twenty-one inches, the second link is the input link at five inches, the third is the coupler at fourteen inches, and the fourth is the output at eighteen inches. The four bar mechanism must rotate between zero and three hundred and sixty degrees at various step values depending on which the user chooses. The initial angular velocity of the input link is 1rad/s and its initial angular acceleration is 1rad/s^2. In the first project only displacement analysis was generated. This project analyzes also the angular velocity and angular acceleration of the four bar linkage.

Methods

In order to properly generate the graphs, correct positions of each of the four links had to be found. This process was done by utilizing vector algebra, derivations, and matrices. The given lengths were inputted into the program and the input link's initial angle, angular velocity, and acceleration were defined in a way that the user can define the desired value in an input dialog box (See Flowchart). The ground link's angle was zero and for the remaining angles, initial guesses were given, but later would change depending up the generated equations. The Newton Raphson equations were used to derive the sum of

the real and imaginary parts of the vector loop. Applying the given constraints and locating

each link's endpoints, equations 1 and 2 were generated [7,5].

$$\Sigma Re = \Sigma x_k s = R_2 cos\theta_2 + R_3 cos\theta_3 - R_1 + R_4 cos\theta_4 \ldots (1)$$

$$\Sigma Im = \Sigma y_k s = R_2 sin\theta_2 + R_3 sin\theta_3 - R_4 sin\theta_4 \ldots (2)$$

The Taylor Series Expansion was then applied to equations 1 and 2. Applying

derivatives with respect to the third and fourth angle in each equation created a Jacobian

matrix in order to linearize the equations (Appendix equations a-e). The inverse of the

Jacobian was next multiplied to equations 1 and 2, which were arranged into a 2x1 matrix

(Appendix equation f). This process also helped adjust the initial guesses made to the third

and fourth links' angles [1,5, 4]. Now the four bar linkage was graphed by defining the start

and end points of each of the four links in the plot function (Appendix Part A and B).

$$\omega_2^2 = \omega_2^2 + 2\alpha_2 d\theta \ldots (3)$$

$$\omega_k = h_k \omega_2, k = 3,4 \ldots (4)$$

After performing the displacement analysis, the angular velocity and acceleration

could be calculated using the kinematic coefficient in addition to the already calculated

matrix in equation f. The kinematic coefficients, h_3 and h_4, were calculated using a

determinant derived in equations g and h in the Appendix. The new values for the third and

fourth link's angular velocity could then be calculated using equation 4. In order to

compute the input link's new angular velocity, equation 3 was used where $d\theta$ is the defined

step size. It was important to make sure all of the equations' units were consistently in

radians and not converted until the end of the program when printing the output values

[6,8].

$$\alpha_k = \dot{h_k}\omega_2^2 + h_k\alpha_2, k = 3,4 \dots (5)$$

The derivatives of the kinematic coefficients were found by taking the inverse matrix in equation i. The new angular accelerations could then be generated by using equation 5. However, for this particular case the initial angular acceleration for the input link was held constant at 1rad/s². The calculated values were then plotted in Figure 3 [6,8].

Results

Using the previously stated design specifications and constraints, the program generated a four bar crank-rocker mechanism, which started at a designated input angle and step value along with initial angular velocity and acceleration. Table 1 gives the generated angles for links 2-4 with an input angle of zero and a step of twenty degrees, which can be verified in the printed values.

Table 1 Angle Outputs based on Figures 2-3		
$\Theta_2(°)$	$\Theta_3(°)$	$\Theta_4(°)$
0	73.40	48.19
20	66.16	53.75
40	58.17	57.07
60	50.85	57.54
80	44.95	55.39
100	40.68	51.31
120	38.10	46.09
140	37.26	40.50
160	38.26	35.22

180	41.17	30.80
200	45.88	27.60
220	52.01	25.75
240	58.98	25.21
260	66.06	25.93
280	72.44	27.90
300	77.20	31.19
320	79.37	35.87
340	78.14	41.77
360	73.40	48.19

Table 2 complements Figure 2 in that it lists the output angular velocity values starting at an initial input of 1rad/s. Due to the calculation performed in equation 5 regarded the input link's angular velocity, a slight increased can be viewed in Figure 2 that is linear. The third and fourth link's angular velocity values intertwine with each other.

Table 2 Angular Velocity Outputs based on Figure 2		
ω_2(rad/s)	ω_3(rad/s)	ω_4(rad/s)
1	-0.31	0.31
1.30	-0.30	0.42
1.55	-0.18	0.47
1.76	0.03	0.48
1.95	0.29	0.45

2.12	0.57	0.37
2.28	0.79	0.24
2.43	0.87	0.03
2.57	0.79	-0.23
2.70	0.52	-0.52
2.83	0.14	-0.75
2.95	-0.27	-0.84
3.06	-0.63	-0.75
3.17	-0.92	-0.49
3.28	-1.13	-0.12
3.39	-1.27	0.29
3.49	-1.34	0.68
3.59	-1.30	0.98
3.68	-1.15	1.15

Unlike the angular velocity of link 2, the input angular acceleration was held constant at 1rad/s^2 throughout the experiment as seen in Table 3 and Figure 3. The third and fourth link's angular accelerations have a similar to curve to their angular velocities in Figure 2. They also intertwine.

Table 3 Angular Acceleration Outputs based on Figure 3		
$\alpha_2(\text{rad/s}^2)$	$\alpha_3(\text{rad/s}^2)$	$\alpha_4(\text{rad/s}^2)$
1	-0.38	0.07

1	-0.12	-0.24
1	0.33	-0.53
1	0.87	-0.70
1	1.41	-0.75
1	1.87	-0.72
1	2.16	-0.58
1	2.35	-0.23
1	2.56	0.43
1	2.82	1.28
1	2.80	1.97
1	2.12	2.23
1	0.76	2.12
1	-0.94	1.90
1	-2.49	1.64
1	-3.50	1.19
1	-3.68	0.28
1	-2.90	-1.21
1	-1.29	-2.98

Figures 1-3 Position, Angular Velocity, and Angular Acceleration Analysis
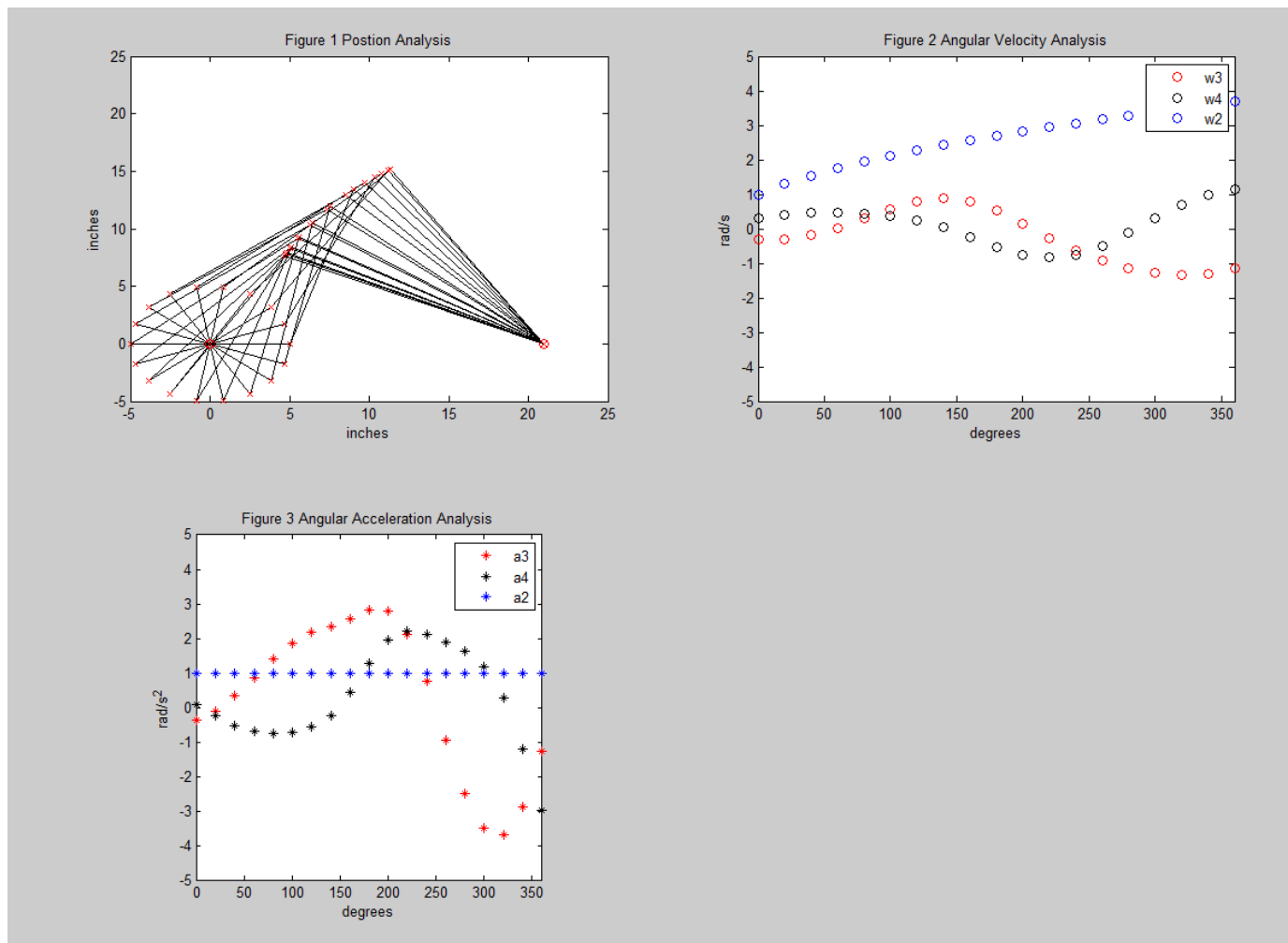
In Figure 1, the mechanism can be viewed rotating from zero to three hundred and sixty degrees. The four bar mechanism's path of motion also confirms that link 1 is fixed and that link 2 is the input. The red circles designate the ground link's endpoints and the red x's designate the joint where two of the links meet (Appendix part A). The program shows the path of motion of each link creating a spiral like shape in Figures 1 [1].. Both Figures 2 and 3 have similar curves, which are expected since the angular acceleration is the derivative of the angular velocity. Note that these figures are relatively small in scale but can be seen better by running the MatLab programs.

Conclusions and Recommendations

The overall design specifications and constraints were clearly stated and able to be executed well in MatLab. Previously learned methods assisted in deriving the equations used to graph the crank-rocker. Generating the rocker and setting up a dialog box for the initial inputs was not difficult. The only tedious part was validating that the equations were derived and coded correctly and that all the units were consistent. However, the program successfully can be called at any input angle, angular velocity, and angular acceleration and at any step value. Depending on which the user prefers, the program can be adjusted accordingly. The time allotted for the project was reasonable and the help sessions were beneficial. The MatLab "tips" sheet did help tremendously. More practice and knowledge of MatLab would help the students become more comfortable with the software.

References

1. Erdman, Arthur G., George N. Sandor, and Sridhar Kota. *Mechanism Design: Analysis and Synthesis.* Vol. 1. 4th Ed. Prentice Hall: Upper Saddle River, 2001.

2. Gouner, Steven, student ME 4133

3. Levert, Laura, student ME 4133

4. Maio, Zhengyu, TA ME 4133

5. ME 4133 Lecture Notes, *Displacement/Position Analysis,* Hall Ch. 1, Erdman & Sandor Ch. 3

6. ME 4133 Lecture Notes, *Vector Loop; Velocity and Acceleration Analysis*

7. Dr. Ram, ME 4133 Fall 2010

8. Dr. Wang, ME 4133 Fall 2011

Appendix I

ME 4133   **Fall 2011**
Machine Design I: Kinematics

Project 2

For the same 4-bar linkage as given for project 1 ((R_1=21in, R_2=5 in, R_3=14in, and R_4=18in), you are required to use the MATLAB software package on PC to generate a plot of the linkage in different positions.

Assume link 1 is fixed as ground. Your program should be able to solve for displacements, velocities and accelerations of the given mechanism for the following inputs

$$\theta_2 = 0° - 360°$$
$$\omega_2 = 1(rad/sec)$$
$$\alpha_2 = 1(rad/sec^2)$$

Please use MatLab to solve for the following:

1) The rotational displacements first (already done in Project 1), the velocities ω_3 for link 3, and ω_4 for the link 4;

2) The accelerations α_3 for the link 3 and α_4 for the link 4 of the mechanism using vector loops and the iterative procedures (Matlab program).

You are required to:

- Write your program as general as possible. You will be graded on this aspect.

- Use your program for project 1 and use it as part of this new project. You should have remembered that you were required to program the project 1 so that it can be used again. Modify your plotting commands from Project 1 to display the data (position, velocity, and acceleration) as functions of the input angle θ_2. Display the linkage in different positions (you may choose a preferred interval).

You are required to use the **kinematic coefficient methods** to solve for the velocity and acceleration. The program should permit the operator to enter different velocity (ω_2) and acceleration (α_2) for link 2.

This project is divided into three sub-projects. After you finish the first and second parts, you ‹should turn in the program and have the TA grade them. After the third part is completed, the complete program and a final project report required.

Due date:

Oct. 28, Friday.

Follow the instructions for project reports as distributed last time.

Flowchart

- Start program with a header and comment description on the purpose of the program
- Define variables
  - Link lengths
  - Accuracy
  - Initial guesses
- Set up dialog box to input initial angle, angular velocity, and angular acceleration (Figure A)
- Define step value (make sure value is in radians)

Figure A
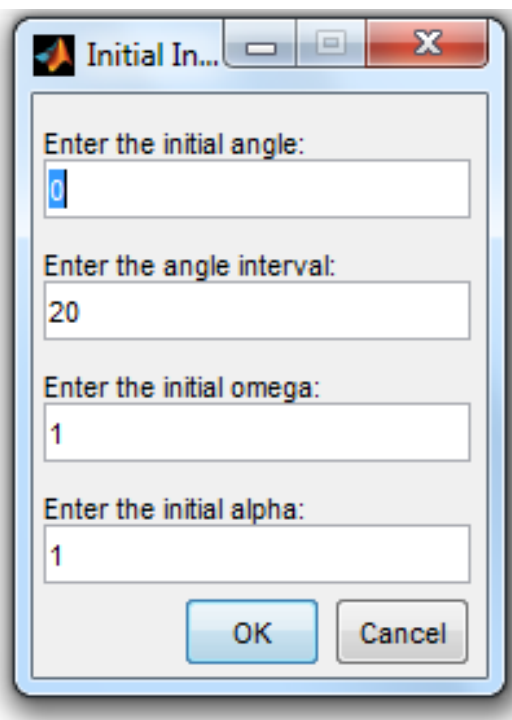
- Initiate 'for' loop
  - Define radius of rotation in radians
  - Set X matrix>accuracy
  - Initiate 'while' loop
    - Should run until accuracy of Newton-Raphson method is acceptable
    - Define matrix with summation of e_1 and e_2 values
    - Jacobian matrix
    - Invert Jacobian
    - Multiply Jacobian with e_1 and e_2 values to calculate the change in angular displacement
    - Calculate new values for the third and fourth links' angles
    - Calculate kinematic coefficients, h_3, and h_4, by taking determinants
    - Calculate new values for third and fourth links' angular velocities by multiplying kinematic coefficient by second link's angular velocity

- Take derivative of 4-bar loop closure equation into kinematic coefficient form arrange into a matrix
- Take inverse of matrix and multiply by original Jacobian
- Calculate new values for third and fourth links' angular accelerations by multiplying the derivative of the kinematic coefficient to the angular velocity of link two raised to the second power and add this value to the product of the kinematic coefficient and the angular acceleration of the second link
- Define vectors for graphing
- Plot
- Define axes, title, axes' labels, resizing, and legends
- Create message box to alert user when the program has successfully ran (Figure B)

Figure B
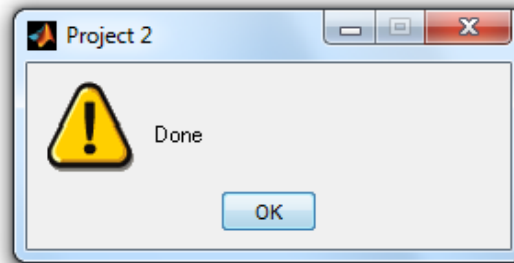
PROGRAM DISCLAIMER

All of these program codes were not original work and they were based on previous knowledge from ME/CSC 2533 and MatLab references. Assistance was also given by TAs, and Dr. Ram's notes. Equations were derived independently with the help of Dr. Wang's notes and the organization of the program was also done independently with some assistance. The programs generate the four bar linkage and its position, angular velocity, and angular acceleration graphs effectively.

Verification of Results

Part A

```
%Kori Lutenbacher
%ME 4133, Fall 2011
%Project number: 2
%Instructor: Dr. Wang
%Created on: October 26, 2011
```

```
%Last Edit: November 2, 2011

%Description: This program solves the position, velocity, and acceleration
%analysis of a four bar linkage

%Link Lengths
r1=21;
r2=5;
r3=14;
r4=18;
rt=pi*180;

%Accuracy
ae=1e-7;
%Initial Guesses
t3=pi/3;
t4=pi/2;
%Input of the initial angle, angle interval, omega, omega interval, alpha,
%and alpha interval
prompt= {'Enter the initial angle:','Enter the angle interval:','Enter the
initial omega:','Enter the initial alpha:'};
dlg_title='Initial Input Values';
num_lines=1;
def={'0','20','1','1'};
answer= inputdlg(prompt,dlg_title,num_lines,def);
%t2=(str2num(answer(1)));
w2=(str2num(answer{3}));
a2=(str2num(answer{4}));
step=(str2num(answer{2}));


for t2=0*pi/180:step*pi/180:360*pi/180
    j=1;
while (norm(j)>=ae)
    %Position Analysis
        e1=r2*cos(t2)+r3*cos(t3)-r1+r4*cos(t4);
        e2=r2*sin(t2)+r3*sin(t3)-r4*sin(t4);
        %Jacobian
        X=[-r3*sin(t3) -r4*sin(t4);
            r3*cos(t3) -r4*cos(t4)];
        f=[e1 e2]';
        b=inv(X);
        j=b*f;
        %new theta values
        t3=t3-j(1);
        t4=t4-j(2);

        %Velocity Analysis Kinematic Coefficients
        h3top=[-r2*sin(t2) -r4*sin(t4);
               -r2*cos(t2) -r4*cos(t4)];
        h3bottom=[-r3*sin(t3) -r4*sin(t4);
                   r3*cos(t3) -r4*cos(t4)];
        h4top=[-r3*sin(t3) -r2*sin(t2);
            r3*cos(t3) -r2*cos(t2)];
        h4bottom=[-r3*sin(t3) -r4*sin(t4);
            r3*cos(t3) -r4*cos(t4)];
        %Determinant
```

```
        h3=(det(h3top))/(det(h3bottom));
        h4=(det(h4top))/(det(h4bottom));
        %new omega values
        w3=h3*w2;
        w4=h4*w2;


        %Acceleration Analysis
        da=[r2*cos(t2)+ r3*(h3^2)*cos(t3)-r4*(h4^2)*cos(t4)
            r2*sin(t2)+r3*(h3^2)*sin(t3)+r4*(h4^2)*sin(t4)];
        dh=b*da;
        dh3=dh(1);
        dh4=dh(2);
        %new alpha values
        a3=dh3*(w2^2)+h3*a2;
        a4=dh4*(w2^2)+h4*a2;




end

%Graph Properties
x1=[0 r1];
y1=[0 0];
x2=[0 r2*cos(t2)];
y2=[0 r2*sin(t2)];
x3=[r2*cos(t2) r2*cos(t2)+r3*cos(t3)];
y3=[r2*sin(t2) r3*sin(t3)+r2*sin(t2)];
x4=[r1 r1-r4*cos(t4)];
y4=[0 r3*sin(t3)+r2*sin(t2)];
pbaspect([1 1 1]);


%Graph Position
subplot 221
xlabel('inches');
ylabel('inches');
title('Figure 1 Postion Analysis');
plot
(x1,y1,'ro',x2,y2,'k',x2,y2,'rx',x3,y3,'k',x3,y3,'rx',x4,y4,'k',x4,y4,'rx');
axis([-5 25 -5 25]);
hold on

%Graph Velocity
subplot 222
plot(t2*180/pi,w3,'ro',t2*180/pi,w4,'ko',t2*180/pi,w2,'bo');
title('Figure 2 Angular Velocity Analysis');
axis([0 360 -5 5]);
xlabel('degrees');
ylabel('rad/s');
hold on
pause(.05);

%Graph Acceleration
subplot 223
plot(t2*180/pi,a3,'r*',t2*180/pi,a4,'k*',t2*180/pi,a2,'b*');
title('Figure 3 Angular Acceleration Analysis');
axis([0 360 -5 5]);
```

```
xlabel('degrees');
ylabel('rad/s^2');
hold on


%Print
fprintf('theta 2 = %3.2f\n', t2*(180/pi));
fprintf('theta 3 = %3.2f\n', t3*(180/pi));
fprintf('theta 4 = %3.2f\n', t4*(180/pi));
fprintf('omega 2 = %3.2f\n', w2);
fprintf('omega 3 = %3.2f\n', w3);
fprintf('omgea 4 = %3.2f\n',w4);
fprintf('alpha 2 = %3.2f\n', a2);
fprintf('alpha 3 = %3.2f\n', a3);
fprintf('alpha 4 = %3.2f\n', a4);

hold on

 %new omega 2
      w2=sqrt((w2^2)+(2*a2*step*pi/180));




end

%legend
subplot 222
legend('w2','w3','w4');
subplot 223
legend('a2','a3','a4');

%The end of a program
h = msgbox('Done','Project 2','warn');
```

Derivations and Analyses

$$\frac{\delta \Sigma Re}{\delta \theta_3} = -R_3 sin\theta_3 \dots a$$

$$\frac{\delta \Sigma Re}{\delta \theta_4} = -R_4 sin\theta_4 \dots b$$

$$\frac{\delta \Sigma Im}{\delta \theta_3} = R_3 cos\theta_3 \dots c$$

$$\frac{\delta \Sigma Im}{\delta \theta_4} = -R_4 cos\theta_4 \dots d$$

$$\begin{bmatrix} -R_3 sin\theta_3 & -R_4 sin\theta_4 \\ R_3 cos\theta_3 & -R_4 cos\theta_4 \end{bmatrix} \dots e$$

$$\begin{bmatrix} -R_3 sin\theta_3 & -R_4 sin\theta_4 \\ R_3 cos\theta_3 & -R_4 cos\theta_4 \end{bmatrix} \begin{bmatrix} \Delta\theta_3 \\ \Delta\theta_4 \end{bmatrix} \dots f$$

$$h_3 = \frac{\begin{vmatrix} -R_2 sin\theta_2 & R_4 sin\theta_4 \\ -R_2 cos\theta_2 & R_4 cos\theta_4 \end{vmatrix}}{\begin{vmatrix} -R_3 sin\theta_3 & R_4 sin\theta_4 \\ R_3 cos\theta_3 & R_4 cos\theta_4 \end{vmatrix}} \dots g$$

$$h_4 = \frac{\begin{vmatrix} -R_3 sin\theta_2 & -R_2 sin\theta_2 \\ R_3 cos\theta_3 & -R_2 cos\theta_2 \end{vmatrix}}{\begin{vmatrix} -R_3 sin\theta_3 & R_4 sin\theta_4 \\ R_3 cos\theta_3 & R_4 cos\theta_4 \end{vmatrix}} \dots h$$

$$\begin{bmatrix} R_2 cos\theta_2 + R_3 h_3^2 cos\theta_3 - R_4 h_4^2 cos\theta_4 \\ R_2 sin\theta_2 + R_3 h_3^2 sin\theta_3 + R_4 h_4^2 sin\theta_4 \end{bmatrix} = \begin{bmatrix} -R_3 sin\theta_3 & -R_4 sin\theta_4 \\ R_3 cos\theta_3 & -R_4 cos\theta_4 \end{bmatrix} \begin{bmatrix} \dot{h}_3 \\ \dot{h}_4 \end{bmatrix} \dots i$$

Lagniappe

Pre-approved enhancements used were the subplot function to generate Figures 1-3, the

legends on Figures 2 and 3, and the input dialog box and message box which validated that

the program ran successfully.